

Massive hybrid stars with a first order phase transition

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We develop our previous study of the transition to deconfined quark phase in neutron stars, including the interaction in the quark equation of state to the leading order in the perturbative expansion within the confinement density-dependent mass model. Using the Gibbs conditions the hadron-quark mixed phase is constructed matching the latter with the hadron equation of state derived from the microscopic Brueckner-Hartree-Fock approximation. The influence of quark interaction parameters on threshold properties and phase diagram of dense neutron star matter are discussed in detail. We find that the leading-order quark interaction expands the density range of the mixed phase, pushing forward the disappearance of the hadron phase. Moreover, since the equation of state could turn out to be stiffer, a high-mass hybrid star is possible with mixed-phase core with typical parameter sets.

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I. INTRODUCTION

With the complementary investigations on heavy-ion collective flows and two precise measurements of heavy mass pulsars [1], the study of neutron stars (NSs) has become a more and more active field of research. *Ab initio* lattice QCD simulations and planned missions LOFT [2] and NICER will certainly promote further our current understanding of the underlying baryonic forces, high-density equation of state (EoS), and NSs' core properties [3]. There might be three kinds of non-nucleonic components in the NS interior: free quarks [4], mesons [5], and hyperons [6, 7]). But no *ab initio* calculations are available so far for their relevance and abundance in NS, because such calculations is unachievable due to the complicated nonlinear and nonperturbative nature of QCD.

In the previous article [4] we have investigated the NS structure within the Brueckner-Hartree-Fock (BHF) approximation, which is currently one of the most advanced microscopic approaches to the EoS of nuclear matter [8]. In that paper BHF was combined with the confinement density-dependent mass model (CDDM) [9] for the quark phase to model hybrid stars (HSs), limiting ourselves to include only the confinement potential in the quark mass scaling. In this work we further extend our calculations by including also short-range leading-order perturbative interactions in the employed quark matter EoS model (i.e., a new version of CDDM model [10]), and explore the consequences for HS structure.

Although in the high temperature region hadron-quark phase transition is crossover as explored by lattice QCD simulations [11], the order of phase transition at zero temperature is still an open problem, as well as the existence of a critical end point (CEP) in the QCD phase diagram. We assume in the present work that the hadron-quark phase transition in cold NSs is a first-order one, and use the Gibbs construction to match the hadron EoS and the quark EoS for obtaining the mixed phase (see Refs. [12, 13] for more discussions on its dependencies on physical situations). That is, the pressure is taken to be the same in the hadron-quark mixed phase to ensure mechanical stability, and would increase monotonically with baryon chemical potential. In the mean time, a global charge neutrality is assumed. Other authors used a smooth crossover [14, 15] to obtain the transition.

We provide a short overview of the theoretical framework and discussions of our results in Sect. II, before drawing conclusions in Sect. III.

II. FORMALISM AND DISCUSSION

A. The hadron phase

Let us first address the hadron phase, that is nuclear matter consisting of nucleons in β -equilibrium with electrons:

$$n \rightleftharpoons p + e^- + \bar{\nu}_e. \quad (1)$$

We omit muons in the following calculations since they are irrelevant to the purpose of the present work. Under the condition of neutrino escape, this equilibrium can be expressed

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as

$$\mu_n - \mu_p = \mu_e, \quad (2)$$

And the requirement of charge neutrality implies

$$n_p = n_{e^-}, \quad (3)$$

where μ_i (n_i) is the chemical potential (the number density) of component i .

The chemical potentials of the non-interacting electrons are obtained by solving numerically the free Fermi gas model. The nucleonic chemical potentials required in Eq. (2) are derived from the energy density of nuclear matter, based on the BHF nuclear many-body approach described elsewhere [16]. Here the input bare nucleon force we employed is the Argonne V18 two-body interaction [17], accompanied by a microscopic three body force constructed from the meson-exchange current approach [18]. The corresponding nuclear EoS reproduces correctly the nuclear matter saturation point and fulfills several requirements from the nuclear phenomenology [19]. We mention here that the model developed in this article misses some important aspects, such as the inclusion of hyperons. The interplay between hyperons and free quarks is quite important and deserves additional investigation, especially when more reliable empirical inputs will be available, especially on the hyperon-nucleon and hyperon-hyperon interaction.

Once the nuclear EoS and the nucleonic chemical potentials of nuclear matter are known, one can then proceed to calculate the composition of the hot β -equilibrium matter by solving Eqs. (2) and (3), together with the conservation of the baryon number, $n_n + n_p = n_B$. Finally the total energy density ε_N and the total pressure p_N of the system are obtained after adding the standard contribution of electrons.

B. The quark phase

The quark phase is considered as a mixture of interacting u , d , s quarks in β -equilibrium with electrons:

$$d \rightleftharpoons u + e^- + \bar{\nu}_e, \quad (4)$$

$$s \rightleftharpoons u + e^- + \bar{\nu}_e, \quad (5)$$

$$s + u \rightleftharpoons d + u. \quad (6)$$

The crucial problem in studying the quark matter is to treat the quark confinement in a proper way. In the framework of the bag model, an extra constant, the famous bag constant B , is introduced which provides a negative pressure to confine quarks within a finite volume. That is, the quark mass is infinitely large outside the bag, and finite and constant within the bag. As is well known, however, particle masses vary from the vacuum to a medium. Taking advantage of the density dependence, one can describe quark confinement without using the bag constant. Instead, the quark confinement is achieved by the density dependence of the quark masses derived from in-medium chiral condensates [9]. That is the CDDM model

we employed in the present study. A large amount of investigation have been performed in the framework of this model, and it has been developed greatly in recent years (see Ref. [10] and references therein).

In the employed CDDM model, strong interactions between quarks are mimicked by an equivalent mass to be determined:

$$\begin{aligned} H_{\text{QCD}} &= H_k + \sum_{q=u,d,s} m_{q0} \bar{q}q + H_I \\ &\equiv H_k + \sum_{q=u,d,s} m_q \bar{q}q \end{aligned} \quad (7)$$

where m_{q0} ($q = u, d, s$) are the quark current mass, H_k is the kinetic term, H_I is the interacting part. The equivalent mass m_q embodies all the interaction effects between quarks. That is, the contributions from both the scalar field and the Lorentz vector field can be included in this way [20].

There are several ways to determine the equivalent mass in the literature (see [21] and references therein). Here we use a recently derived mass formula at zero temperature [10]:

$$m_q \equiv m_{q0} + m_I = m_{q0} + \frac{D}{n_B^{1/3}} + C n_B^{1/3}. \quad (8)$$

where m_I is the interacting mass, parameterized as a function of the baryon number density n_B . D term is derived from the non-perturbative linear confinement of quarks (see also [9]), and C term comes from the short-range leading contribution of perturbative interactions. These two terms correspond to the two leading terms in both directions when expanding the equivalent mass to a Laurant series of the holistic Fermi momentum, respectively [10]. The confinement interaction dominates at lower densities, while the perturbative interactions becomes more important at higher densities. This model gives reasonable results for the sound velocity. Also, strange quark matter in bulk still has the possibility of absolute stability for a wide range of parameters. As to the quark current mass, in our calculations we take $m_{u0} = m_{d0} = 0$ and $m_{s0} = 95$ MeV.

The parameter D has a lower bound $D^{1/2} = 156$ MeV, and an upper bound $D^{1/2} = 270$ MeV [22]. The lower bound comes from the nuclear physics constraint, demanding that at $P = 0$, non-strange nuclear matter should be stable against decay to (ud) quark matter. This leads to the condition $E/A > M_{56\text{Fe}} c^2 / 56 = 930$ MeV for (ud) quark matter, which gives the above mentioned lower bound. The upper bound can be derived from a relation between D and the quark-condensate and the known range of values for this condensate [22]. The upper boundary of 270 MeV is in fact a very conservative one. According to the updated quark condensate determined nowadays very precisely by lattice QCD [23], a range of (161 MeV, 195 MeV) can be obtained. Therefore in this work, we take two typical values of the confinement parameter as $D^{1/2} = 170$ MeV, 190 MeV as inferred by the newest lattice QCD results [23].

The parameter C depends on how the strong coupling runs and it is determined so to have an upper bound of $C = 1.1676$ [10]. Previous calculations [10] of pure quark stars employing a CDDM EoS lead to a maximum mass as high as $2M_\odot$ with a parameter set $(C, D^{1/2}) = (0.7, 129 \text{ MeV})$. We

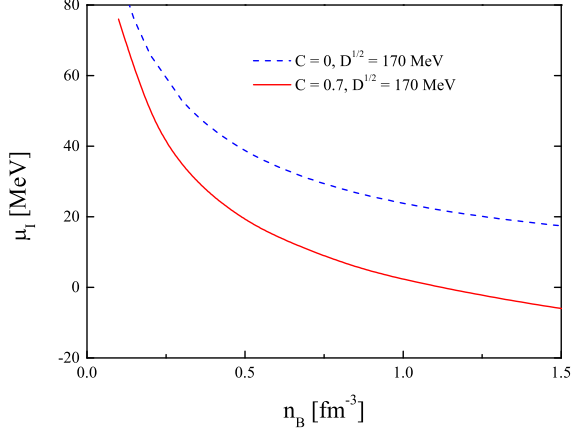


FIG. 1: (Color online) Extra chemical potential as a function of the baryon density, for both $C = 0$ and $C = 0.7$ cases with $D^{1/2} = 170$ MeV.

will then employ $C = 0.7$ to perform calculations and change its value in a certain range for comparison.

The relevant chemical potentials μ_u , μ_d , μ_s , and μ_e satisfy the weak-equilibrium condition (we again assume neutrino escape):

$$\mu_u - \mu_d = \mu_e, \quad \mu_d = \mu_s. \quad (9)$$

The baryon number density and the charge density can be written as

$$n_B = \frac{1}{3}(n_u + n_d + n_s), \quad (10)$$

$$qQ = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e. \quad (11)$$

The charge neutrality condition requires $qQ = 0$.

Since the quark masses m_i are density dependent, the quark chemical potentials μ_i have an additional term μ_l with respect to the free Fermi gas model ($j = u, d, s$):

$$\mu_i = \frac{\partial \varepsilon_i}{\partial v_i} \frac{dv_i}{dn_i} + \sum_j \frac{\partial \varepsilon}{\partial m_j} \frac{\partial m_j}{\partial n_i} = \sqrt{v_i^2 + m_i^2} - \mu_l \quad (12)$$

The quark energy densities are

$$\varepsilon_i = \frac{3}{\pi^2} \int_0^{v_i} \sqrt{p^2 + m_i^2} p^2 dp, \quad (13)$$

$$\varepsilon = \sum_i \varepsilon_i, \quad (14)$$

where $v_i = (\pi^2 n_i)^{1/3}$ are the Fermi momenta and $\partial m_j / \partial n_i$ are derived from Eq. (8) by taking the derivative of the baryon density. The quark pressure is calculated as $p = -\varepsilon + \sum_i \mu_i n_i$. Solving Eqs. (9), (10) and (11), the total energy density ε_q and pressure p_q of the system can be obtained after adding the contribution of the leptons.

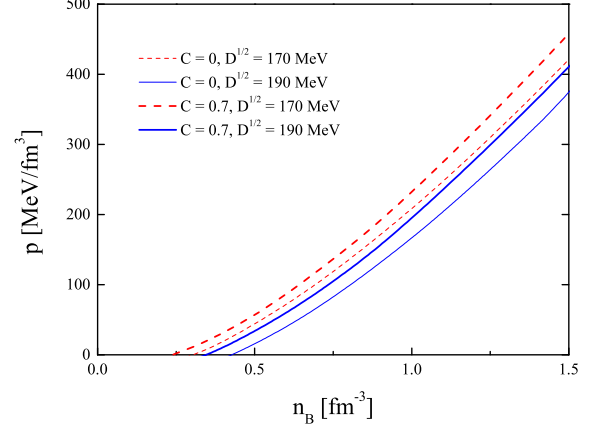


FIG. 2: (Color online) Pressure of β -stable quark matter as a function of baryon density for two values of $C = 0, 0.7$, and two values of $D^{1/2} = 170$ MeV, 190 MeV.

In our previous calculations [4] there is only the term D in the quark mass scaling, which results in relatively low quark thresholds and small HS masses, similar to the calculations [24] within the color dielectric model and the MIT bag model. As will be shown later, inclusion of the C term may bring strongly repulsive quark interactions, pushing the quark matter to appear at appropriately high densities, and consequently making massive HSs in the model.

For a better understanding of this point, we present in Fig. 1 the modification of the extra chemical potentials μ_l induced by the nonzero perturbative parameter C . The calculations are done with fixed $D^{1/2} = 170$ MeV and two values of $C = 0, 0.7$. We see that with increasing density, μ_l always decreases. A nonzero C makes it decrease faster with the density. The decrease is even more pronounced at higher densities, because the term $C n_B^{1/3}$ is an increasing function of the density. Also, the extra chemical potential μ_l changes from positive to negative values at high densities for $C = 0.7$, indicating that the term C brings repulsions, and quarks in this case are more strongly interacting with each other. This effect of C is opposite to that of D , since the latter arises from the quark confinement potential, and the increase of this parameter will bring attractions and soften the EoSs of the matter. Fig. 2 clearly demonstrates these effects, where the EoSs of β -stable quark matter are shown for the two values of $C = 0, 0.7$, and the two values of $D^{1/2} = 170$ MeV, 190 MeV. The increase of the confinement parameter D will soften the EoS, while the perturbative parameter C will stiffen it. The repulsive nature of the term C will have crucial consequences for the structure of the resulting HSs as seen later.

C. The mixed phase

Let us consider the mixed phase made of nucleon matter in equilibrium with a gas of u, d, s quarks and electrons. Assum-

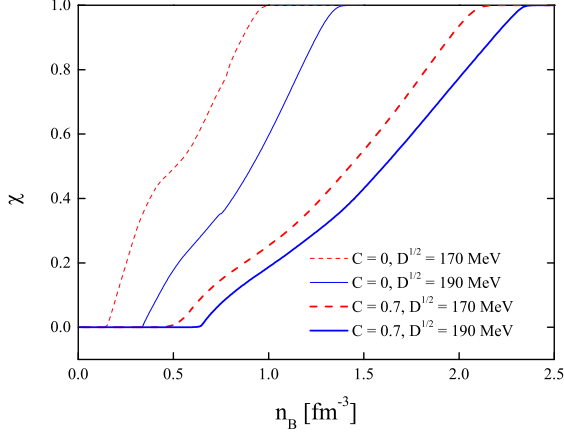


FIG. 3: (Color online) Quark fraction χ as a function of the baryon density, for two values of $C = 0, 0.7$, and two values of $D^{1/2} = 170$ MeV, 190 MeV.

ing the global charge conservation, the conservation laws can be imposed introducing the quark fraction χ defined as

$$\chi \equiv V_q/V, \quad (15)$$

where V is the total volume and V_q the volume occupied by quarks. In terms of χ the total baryon density, total electric charge and total energy density are written

$$n_B = (1 - \chi)n_N + \chi n_q, \quad (16)$$

$$Q_t = (1 - \chi)Q_N + \chi Q_q, \quad (17)$$

$$E_t = (1 - \chi)E_N + \chi E_q, \quad (18)$$

respectively. The quantities $n_N(n_q)$, $Q_N(Q_q)$, and $E_N(E_q)$ are nucleonic (quark) number density, charge density, and energy density, respectively.

Nucleonic chemical potentials are connected to quark chemical potentials as follows:

$$\mu_n = \mu_u + 2\mu_d, \quad (19)$$

$$\mu_p = 2\mu_u + \mu_d. \quad (20)$$

Therefore, there are only two independent chemical potentials. For a given total density n_B , the two independent chemical potentials and the quark fraction χ can be determined by solving the charge neutrality equation $Q_t = 0$ and the pressure balance equation $p_N = p_q$.

In Fig. 3, the quark fraction χ is plotted as a function of the baryon density, for the two values of $C = 0, 0.7$, and the two values of $D^{1/2} = 170$ MeV, 190 MeV. As already known from our previous work, the larger D value pushes the threshold of quark matter phase to higher densities in the case of $C = 0$. This conclusion still holds when we adopt a nonzero value

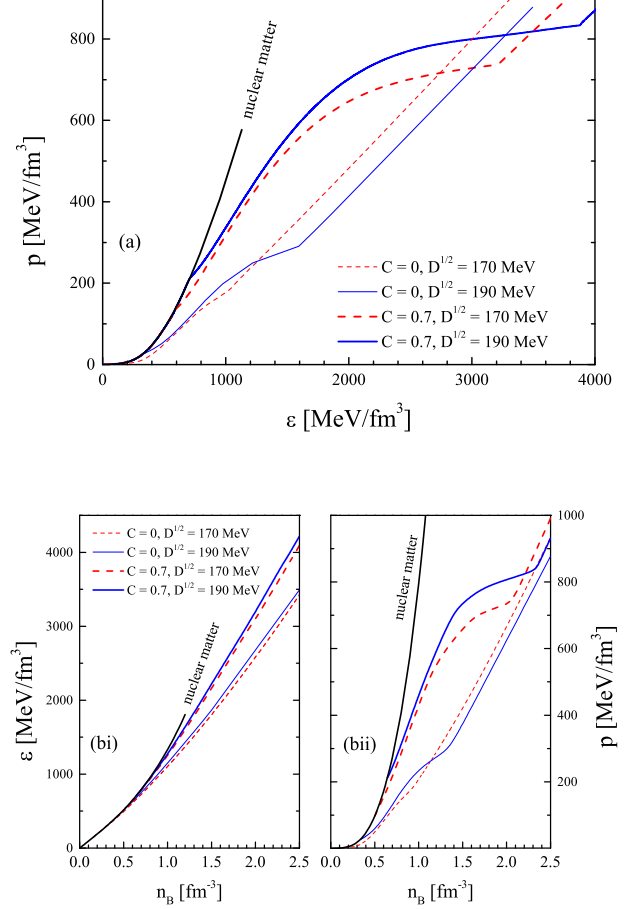


FIG. 4: (Color online) (Upper panel) Pressure of HS matter as a function of energy density and (lower panel) energy density and pressure of HS matter as a function of baryon density, for two values of $C = 0, 0.7$, and two values of $D^{1/2} = 170$ MeV, 190 MeV. The results of pure nuclear matter are also shown for comparison.

of C in the quark mass scaling, increasing the critical density from 0.57 fm^{-3} to 0.64 fm^{-3} at $(C, D^{1/2}) = (0.7, 190 \text{ MeV})$. Also, a nonzero positive C is found to have the same effect, when the $C = 0.7$ cases are compared with the corresponding $C = 0$ cases. A critical density 0.16 fm^{-3} (0.33 fm^{-3}) with $C = 0, D^{1/2} = 170$ MeV (190 MeV) is increased to 0.57 fm^{-3} (0.64 fm^{-3}) with $C = 0.7, D^{1/2} = 170$ MeV (190 MeV).

In turn, when the C value is chosen to be negative, that is allowed by our model, quarks appear even earlier than the $C = 0$ case. This means that for typical model parameters free quarks could be present below the nuclear saturation density 0.16 fm^{-3} , which is unphysical, so that we have to exclude negative C values in the present work. Actually, a previous study showed that negative C values were allowed, for a simple inclusion of one-gluon-exchange interaction between quarks and degeneration 4 with respect to spin and flavor [25]. This can be understood as follows: Since we start from the quark potential, the spin and flavor degrees of freedom should

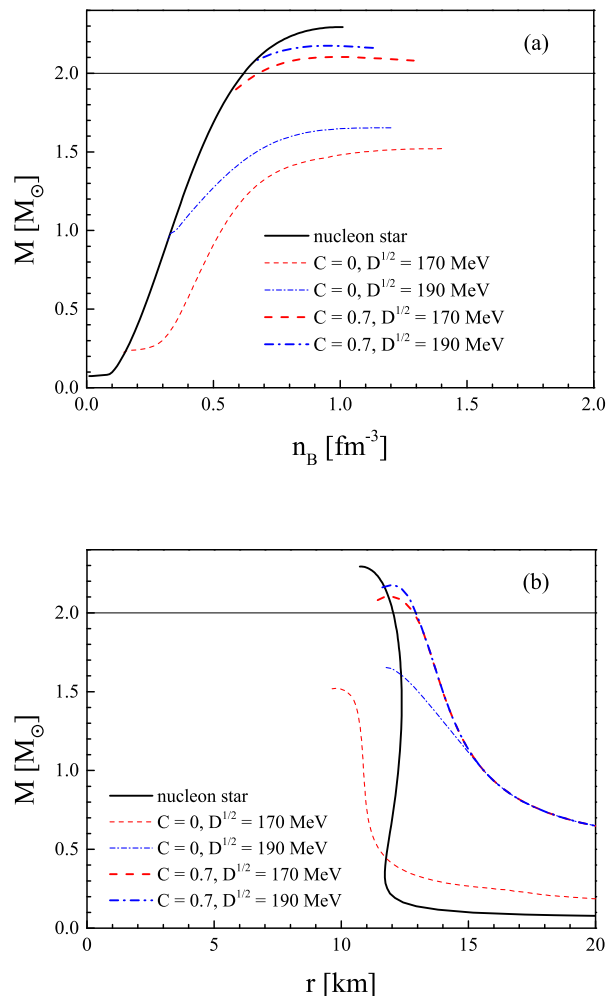


FIG. 5: (Color online) (Upper panel) HSs' masses as a function of their central densities and (lower panel) HS mass-radius relations, for two values of $C = 0, 0.7$, and two values of $D^{1/2} = 170$ MeV, 190 MeV. The results of the nucleon star are also shown for comparison, as well as the recent 2-solar-mass constraint from the mass measurements of PSR J1614-2230 and PSR J0348+0432 [1].

be very important. Especially, the spin-spin interaction between the quarks plays an important role in calculating the effective repulsion [26]. A more realistic inclusion of the quark potential as done in [27] could then lead to a more reasonable result of the critical density around 0.5 fm^{-3} . Furthermore, when the term C is included, together with an upper quark threshold, the density range of the mixed hadron-quark phase is expanded (around twice with the chosen parameters). This should be a general result when the Gibbs construction is employed to achieve a first-order phase transition.

The stable configurations of a NS can be obtained from the well-known hydrostatic equilibrium equations of Tolman, Oppenheimer, and Volkov [28]. At variance with pure nuclear or quark stars, a HS may contain pure quark matter in the core, pure nuclear matter near the outer part, and, in between, a

mixed phase of the quark and nuclear matter. We then employ corresponding EoS models described above. They are shown in Fig. 4, where the pressures of HS matter as a function of energy density are shown in the upper panel, and the energy densities and pressures of HS matter as a function of baryon density are shown in the lower panel, for two values of $C = 0, 0.7$, and two values of $D^{1/2} = 170$ MeV, 190 MeV. These plots show again that the term C can stiffen the EoS, pushing the occurrence of free quark phase deeper into the star core, as discussed above. For the description of the NS's crust, we have joined the hadronic EoSs above described with the ones by Negele and Vautherin [29] in the medium-density regime ($0.001 \text{ fm}^{-3} < \rho < 0.08 \text{ fm}^{-3}$), and the ones by Feynman-Metropolis-Teller [30] and Baym-Pethick-Sutherland [31] for the outer crust ($\rho < 0.001 \text{ fm}^{-3}$).

Fig. 5 shows the corresponding HSs' masses as a function of their central densities (upper panel) and also the HS mass-radius relations (lower panel) with the chosen parameters. The results of the nucleon star are also shown for comparison. We see that a higher mass is achieved when the term C is included, for example, the value of $1.53M_\odot$ ($1.65M_\odot$) of the maximum mass obtained with $C = 0, D^{1/2} = 170$ MeV (190 MeV) is increased to $2.10M_\odot$ ($2.17M_\odot$) using $C = 0.7, D^{1/2} = 170$ MeV (190 MeV). Only in the case of $C = 0, D^{1/2} = 170$ MeV, a pure quark core can be reached, while in other three cases, the most massive stars both have a mixed-phase core. We have also checked that, an even larger C value will bring heavier HSs, but always with a mixed-phase core. This means that no pure core is possible in the present HS model and quarks only appear in a limited region of the NS's core. These results are consistent with a latest study using the chiral effective field theory approach joined with the Polyakov-Nambu-Jona-Lasinio (PNJL) model [32].

III. CONCLUSIONS

Summarizing, we have presented updated calculations of the transition from hadron to quark deconfined phase in NS matter, and also the HS structure based on our previous work. This extension concerns mainly the quark matter EoS, where we used a recent derivation of the quark mass scaling, including the leading-order perturbative interactions, in addition to the quark confinement. The derivation scheme allows us to modify largely the high-density behaviour of dense matter, resulting from a more repulsive quark interaction.

We find that the quark thresholds are pushed to high densities, together with large density jumps in the first order phase transition. Also, the EoSs are stiffened and the resulting HS maximum mass are shifted to higher values. Massive HSs as high as $2M_\odot$ are possible, consistently with two recent astrophysical observations of pulsars in binary systems.

In the near future we plan to include the color superconductivity since it is expected to play a role in dense quark matter at the density range discussed in the present work. Also, the appearance of hyperons, missing in our present version of BHF model, could be studied in competition with free quarks, since a previous study [33], combining the same nucleon model

with the Dyson-Schwinger quark model shows that no hybrid star can exist if hyperons are introduced. Finally, we would like to study how such high-mass NSs are formed in a binary system.

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